THE ROLE OF HANDHELD TECHNOLOGY IN TEACHING AND LEARNING SECONDARY SCHOOL MATHEMATICS

Gail Burrill, Michigan State University, USA
burrill@msu.edu

Students learn mathematics by doing mathematics, engaging in tasks and activities, mediated by the teacher. Technology's influence on students' mathematical learning is either amplified or limited through the kinds of mathematical tasks and activities teachers provide. The newest generation of handheld technologies can provide unique opportunities for students to do mathematical tasks in new ways that have the potential to foster learning and develop understanding.

Many of the graphing calculators and computer algebra systems in use today are tools for computation or displaying mathematical relationships in a variety of representations. In particular, graphing calculators have had an effect on the mathematics curriculum in secondary schools in the United States, making popular the rule of three – graphs, tables and symbols- as ways to represent and analyze relationships. Dynamic geometry computer software such as Geometer's Sketchpad or Cabri allows students to manipulate geometric objects and see continuous updates of measurements and constructions. The research on the appropriate use of both of these technologies suggests that they can have a positive influence on what and how students learn mathematics. For the work on handheld graphing technology, see for example, Burrill, et al, 2002; Ellington, 2003, 2006; Graham & Thomas, 2000; Schwarz & Hershkowitz, 1999; Hollar & Norwood, 1999. Dynamic geometric software helps students explore, conjecture and explain geometric relationships, serves as a basis for developing understanding of proof, and it use can produce measurable learning gains (Jones, 2002; Hollebrands et al, in press). However, the research also makes clear that what is important is how both of these technologies are used in classrooms by teachers.

New technologies such as TI-Nspire bring together both of these environments in one handheld, providing the opportunity to create an even wider variety of dynamic linked representations, where a change in one representation is immediately and visibly reflected in another. For example, a user can manipulate the graph of a function by grabbing and dragging and immediately see the resulting changes in the algebraic form. Dynamic links among spreadsheets, graphing environments, geometry settings, and symbolic expressions allow students to take meaningful actions on mathematical objects and immediately see the consequences of those actions. Together with a document structure similar to folders in a word processing package, this new technology enables students to enter an environment where they can explore mathematical concepts in new and deep ways.

To exploit this dynamic environment, just as with handhelds and dynamic geometry software, students need to have adequate opportunities to conjecture, reflect, explain, and justify. Thus, what the teacher asks students to do and to think about is critical if the technology is to be a tool for learning mathematics as well as doing mathematics. And much of what teachers ask students is based on the materials they use.

The design of technology-based activities for learning mathematics, however, needs careful consideration. There is a real danger that such materials will fall into categories absent any emphasis on what mathematical learning they will enable. For example, materials such as the following are likely to appear (adapted from Belfort & Guimaraes (2004): 1) the author's interest is on mastering the use of the technology where the mathematics is secondary; 2) the activity is merely a demonstration of an idea where students are treated as spectators; 3) the activity revisits a mathematical topic to show how it can be done in a simple way with the new technology where the students' role is verification; 4) the author replicates activities from the point of current instructional materials, underestimating the technology's potential, where the ideas are fragmented and obtaining a formula is often the objective. Materials developed for TI-Nspire can suffer from these same pitfalls: construction of the mathematical objects involved in the problem can be the focus along with all of the details needed to master the device; elaborate constructions demonstrate a relationship but allow no interaction on the part of students, the device is used to check answers rather than investigate what answers might be possible and why; activities replicate what can be done on a graphing calculator with no attention to the opportunities afforded by the new capabilities.

Some guiding principles can facilitate the development of materials and activities for TI-Nspire that takes advantage of the potential of the device to enhance mathematical learning (Dick et al, 2007):

- Activities should have a clear focus on important mathematical ideas.
- Activities should allow students to deliberately take mathematically meaningful actions on objects and to immediately see the mathematically meaningful consequences of those actions.
- Activities should include inquiry tasks of high cognitive demand.

Even with the framework of these principles, the almost unlimited opportunities provided by the device (currently five different applications that can be connected in multi-dimensional ways: graphs and geometry, calculator, lists and spreadsheets, data and statistics, and notes) are daunting. While many teachers (those technology inclined) are excited by these possibilities, many others are overwhelmed by the knowledge needed to understand how to use all of the applications. Participant evaluation data from TI-Nspire workshops include comments such as the device is far too complex for most students, which in fact, is probably not true but instead is a reflection of the teachers' own insecurity with learning to operate the device. Given this perception and beliefs on the part of teachers and given the premise that TI-Nspire can help more students learn more and better mathematics, we need to look carefully at the materials we put into the hands of teachers.

The view that my colleagues, Wade Ellis and Tom Dick, and I have taken is that by imposing constraints on what is possible, teachers and students will actually have more freedom to explore important mathematical concepts in deeper ways. Thus, we are looking to the creation of "microworlds" in which students can play with a mathematical idea in a variety of ways but where the opportunity to go astray, both mathematically and operationally, is limited. A microworld is similar to an applet, but many applets are designed for a specific outcome (e.g. the applets developed at the Freudenthal Institute http://www.fi.uu.nl/wisweb/en/). We envision a microworld to have several general applications related to a concept and grounded in developing conceptual understanding. As our notion of microworlds evolves, these worlds seem to have certain characteristics.

- They require very little knowledge on the part of the user of the device itself and how it operates.
- The fundamental idea is simple and straightforward. The development has both mathematical fidelity (is mathematically sound and accurate) and pedagogical fidelity (does not present obstacles such as cluttered screens or too many decimal places that interfere with learning) (Dick & Burrill, 2006).
- The design is based on the action consequence principle described above, taking a mathematical action and immediately seeing the consequences.
- The "world" is based on a template that usually has one object such as a point, line, shape or graph serving as a driver for the interaction.
- Using this template, a series of learning activities can be constructed either by the classroom teacher or another author that will enable teachers to adapt this "world" for their particular mathematical objective.
- The world is based on important mathematics.

MICROWORLDS

One example of a microworld might be a template that contains a line and a slope triangle (the triangle formed by using the change in *y* and the change in *x* for two points on the line) with the formula for slope on screen and linked to the triangle (Figure 1). The mathematical action is to drag one of the points on the line or the line itself; the consequences are changes (or not) in the lengths of the sides of the triangle, the numerator and denominator in the slope formula and to the slope. Given this template, a variety of explorations and appropriate inquiry questions can be created that will allow students to investigate slope dynamically, where they cause changes and follow up on the consequences.

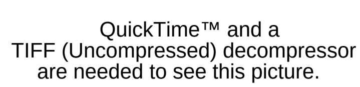

Figure 1: Slope triangle

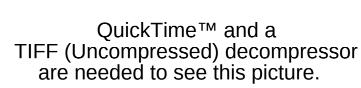

Figure 2: Algebraic expressions

Another example is a template that has a number line with a point that displays integer values as you drag it (Figure 2). Algebraic expressions in text boxes visible on the screen are linked to the point. As you drag the point, the values for the expressions appear under the expressions. The questions can engage students in thinking about variables, evaluating expressions, solving equations (Will the expression ever be equal to 1?), inequalities (For what x will the first expression generate values that are less than the second? Why?), and linearity (How does the output change for a given change in *x*? Is there any regularity?)

With small changes, the same template can be used in multiple settings. Figure 3 shows the same basic template concept but using two variables. The questions might focus on equality, inequality, rates of change, or systems of equations.

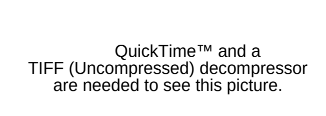

Figure 3: Pairs of expressions

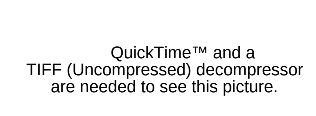

Figure 4: Plotting the solutions

On a second page, the teacher may choose to collect the values that make the expressions equal in a spreadsheet and plot them (Figure 4). The questions can push students to consider what relationship exists among the values that make the two expressions equal and what in the expressions themselves might account for this. Students can change the expressions on their own or at the teacher's direction by clicking on the text box and changing a single value or rewriting the entire expression.

The action-consequence principle where one object "drives" another has been useful in thinking about classes of templates, depending on the objects. The examples above are based on changing a point to drive a change in the numerical result of an expression. Figures 5 and 6 show how changing a point drives a change in the shape to investigate the properties of a parallelogram that are essential in computing area.

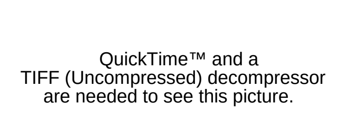

Figure 5: Area of square

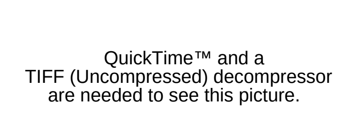

Figure 6: Area of parallelogram

Another example could use a graph as a driver, for example where a change in one graph causes change in another graph, i.e., graphing a function and its derivative.

IMPLEMENTATION

Our immediate work has been in response to content areas that have emerged in a study of high stakes state assessments (e.g., end of course or high school exit exams) as areas in which student achievement is low. Such areas include reasoning about concepts and relationships; perimeter, area and volume; and manipulation of algebraic expressions (Burrill & Dick, 2007). The work is really in the first stages. We have designed several templates and have used them in a variety of settings with teachers who primarily been technology users, i.e., workshops, conferences, focus sessions, working groups. They have been used in two university courses: a mixed undergraduate and graduate (which included practicing teachers) course and a preservice course for prospective secondary mathematics teachers, where the backgrounds ranged from experienced technology users to minimally proficient. The initial reactions seem to be of two kinds, enthusiasm and willingness to design lessons around the templates that take advantage of the opportunities it provides and concern about how to use the templates effectively where some of the lessons offered do not get at the deeper learning issues in a substantive way. A more detailed analysis will be conducted at the end of the semester.

The input thus far has raised some critical issues that need to be addressed as we move forward. One relates to the actual design of the template to maximize learning. Just as in Japanese lesson study (see Makoto, 2002, for example), where seemingly minute features of a lesson are carefully examined for their contribution to developing understanding, the design of the templates has to be equally as carefully done if using the template is to enable learning. For example, one of the areas identified as problematic for students was dealing with problems involving percents. The template in Figure 7 was the first attempt, where dragging either point changes the base, the part and the percent. It is also possible to change the unit or the scale in the upper left corner.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Figure 7: Percent segments

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Figure 8: Percent as area

However, the design raised questions for those using it. The undergraduates and graduate students as well as some teachers wanted to change the order of the segments, putting the segment representing the whole under the segment representing the part. One of our colleagues (Arnold, 2008) suggested an alternate approach, where percent is represented as area (Figure 8). Modifying the design is not something to be done lightly, as each option might have real implications for learning. Because the usual way to set up a fraction is with the part in the numerator and the whole in the denominator, is that the more intuitive way for students? Or is it actually more sense-making for a student who has not had this prior knowledge to begin with the whole, leaving it above the segment representing the part. Does using an area model bring conceptual disadvantages that might interfere with later learning? Favoring one of the three versions over the others raises these and other questions.

Other design issues relate to using discrete vs. continuous values in certain settings; for example, will having a continuous version of the number line template in Figure 2 be useful. Does the placement of labels make a difference; in Figure 8, should the 40% be inside the rectangle? Should the complement be there as well? How much detail on the screen is important for developing understanding of a concept; i.e., in Figure 5 should the formula at the top say $b \cdot h = 15.354 \cdot 10.378u^2 = \text{area}$? Thus, part of our work is to frame research around such questions that will inform the design of specific templates and also begin to give us general guidelines for continuing the development of templates for a broader set of mathematical concepts.

The trial work with teachers also made clear that at some level, they needed to understand the reasons why the design was as it was. In some cases, teachers modified the template according to their notions and removed an important pedagogical element (the angle measures in Figure 5 and 6) or created possibilities for misconceptions (removing the measure of the base in Figure 5). Thus, if teachers are to maximize the templates as learning devices, some scaffolding has to be provided that will enable them to make sense of how the design contributes to student learning.

A second issue related to the use of the templates is also connected to implementation: how to help teachers learn to ask questions that probe at student thinking and open up opportunities for learning. If the tool is to be one for learning mathematics as well as doing mathematics, students need to respond to questions that ask them to conjecture (what if…, how could …), reflect (why did…, how are they alike…), explain, and justify. Thus, teachers need to design lessons using the templates mindful of those goals and questions that will lead to those goals. Framing such inquiry questions is not easy: in our first attempt at posing questions for the percent template, we realized that we had focused on narrow closed questions. The educators with whom we shared our next iteration challenged some of our questions. In the workshops, teachers often moved rapidly through a template, dragging the point or the graph and commented on what they observed. Thinking about the questions they might have students consider when they used the template was difficult for them. A major challenge will be provide support for teachers to learn how to pose such questions and manage the responses.

Based on our limited experience, the templates do seem to open avenues for student thinking and create opportunities for discussion not only about the mathematics but about strategies for using the template to reason about the mathematics. For example, some teachers made deliberate changes in what they can move and record the results; Using the number line template (Figure 3), some held the x-value constant, increased the y-value by 1 unit at a time, and recorded the output then changed the x-value by 1 and repeated the process; others looked at extremes, what happens to the difference between the expressions when x is large or when x is small. Helping students understand how to think in these ways is a fundamental part of learning to do mathematics, and it will be important for teachers to recognize this.

The next phase of the work is to formally pilot the templates and to consider the implications for research related to both the design and implementation. The potential is clearly present to make a difference in what students learn; the challenge is to make this happen in ways that can be replicated across the teaching community.

References:

Belfort, E. & Guimaraes, C. (2004). Teachers' practices and dynamic geometry. In Hoines, M. & Fuglestad, A. (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*. (2, pp. 503-510).

Burrill, G., Breaux, G., Kastberg, S., Leatham, K., & Sanchez, W. (2002). *Handheld graphing technology at the secondary level: Research findings and implications for classroom practice*. Dallas, TX: Texas Instruments Corp. http://education.ti.com/research

Burrill, G. & Dick, T. (2007). What state assessments tell us about student achievement in algebra. (in preparation)

Dick, T., Burrill, G. & Brady Gill, L. (2007). New technologies offer new ways to engage students. NCSM Newsletter, (38,1). Pps.19-21

Dick, T. & Burrill, G. (2006) Presentation at National Council of Supervisors of Mathematics Annual Meeting, Atlanta GA.

Ellington, A.J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal of Research in Mathematics Education*. (34, 5).

Ellington, A. J. (2006) The Effects of non-CAS graphing calculators on student achievement and attitude levels in mathematics: A meta-analysis. *School Science and Mathematics*

Graham, A. T., & Thomas, M. O. J. (2000). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics, 41*(3), 265-282.

Hollar, J. C., & Norwood, K. (1999). The effects of a graphing-approach intermediate algebra curriculum on students' understanding of function. *Journal for Research in Mathematics Education, 30*(2), 220-226.

Hollebrands, K., Laborde, C., & Straber, R. (in preparation). The learning of geometry with technology at the secondary level. In Heid, K. & Blum, G. (Eds.), *Handbook on Research on Technology in the Learning and Teaching of Mathematics: Mathematics Curriculum Development and Tool Development*. Greenwich CT: Information Age.

Jones, K. (2002). Implications for the classroom: Research on the use of dynamic software. *Micromath.* http://eprints.soton.ac.uk/14689/

Schwarz, B. B., & Hershkowitz, R. (1999). Prototypes: Brakes or levers in learning the function concept? The role of computer tools. *Journal for Research in Mathematics Education, 30*(4), 362-389.

Yoshida, M. (2002). Framing lesson study for U.S, participants. In Bass, H., Usiskin, Z., & Burrill, G. (Eds.), *Studying classroom teaching as a medium for professional development. Proceedings of a U.S.-Japan workshop*. Washington DC: National Academy Press.